# Philadelphia University

## Department of Basic Sciences


**Final Exam** **Number Theory** **02–02–2014**

Write complete solution for each problem.

1. Suppose that $\gcd(m, n) = 1$. If $m \mid k$ and $n \mid k$, prove that $mn \mid k$.
2. Count how many divisors for the number 151200.
3. Use Wilson's theorem to compute $33!\ \%\ 37$.
4. Use Euler's theorem to compute $3^{17929}\ \%\ 11600$.
5. Suppose that $\gcd(a, 35) = 1$. Use the Chinese remainder theorem (CRT) to prove that $a^{12} \equiv 1 \pmod{35}$.
6. The number $g = 3$ is a primitive root modulo 17. Find all the primitive roots modulo 17.
7. Solve the discrete logarithm problem $4^x \equiv 2 \pmod 7$ using the fact that $g = 3$ is a primitive root modulo 7.
8. Find all the quadratic residues (QR) and non-residues (NR) modulo 13.
9. Evaluate the Legendre symbol $\left(\dfrac{909}{1567}\right)$.
10. Use the Chinese remainder theorem (CRT) to solve the quadratic congruence $x^2 \equiv 26 \pmod{55}$.

–Amin Witno

The list of prime numbers $p < 200$:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 |
| 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 |
| 73 | 79 | 83 | 89 | 97 | 101 | 103 | 107 | 109 | 113 |
| 127 | 131 | 137 | 139 | 149 | 151 | 157 | 163 | 167 | 173 |
| 179 | 181 | 191 | 193 | 197 | 199 | | | | |