Final Exam **Number Theory** **03–02–2016**

1. (7 points) Find all the integer solutions (congruence classes) for the congruence $x^{13} \equiv 2 \pmod{23}$.

2. (7 points) Find all the integer solutions (congruence classes) for the discrete logarithm problem $2^x \equiv 15 \pmod{17}$ using the primitive root $g = 3$.

3. (5 points) Use the Chinese remainder theorem and Fermat's little theorem to prove that $n^{17} \equiv n \pmod{85}$ for all integers n.

4. (4 pts) Use Gauss' theorem (lemma) to evaluate the Legendre symbol $\left(\dfrac{6}{23}\right)$.

5. (6 pts) Use Legendre symbol to prove if there is a solution or no solution for the quadratic congruence $2x^2 - 6x + 79 \equiv 0 \pmod{113}$.

6. (6 pts) Find all the integer solutions (congruence classes) for the quadratic congruence $x^2 \equiv 71 \pmod{77}$.

7. (5 pts) Let g be a primitive root mod p, a prime nnumber. Prove that $\left(\dfrac{g}{p}\right) = -1$.

–Amin Witno

The list of primes below 200.

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199				