

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2

Number Theory

09–05–2018

1. (2 points) Count how many primitive roots exist mod 409. (Note: 409 is prime.)
2. (2 points) Let a be a primitive root mod 17. Compute $|a^{20}|_{17}$.
3. (3 points) Use Successive Squaring Algorithm to compute $2^{77} \% 100$.
4. (3 points) Prove that $n^{45} \equiv n \pmod{115}$ for all integers n , using the Chinese remainder theorem and Fermat's little theorem. (Note: 115 is composite.)
5. (3 points) Solve the congruence $x^{19} \equiv 5 \pmod{36}$.
6. Use your solution of Problem (a) to solve (b) and (c).
 - (a) (2 points) Prove that 7 is a primitive root mod 22.
 - (b) (2 points) Find all the primitive roots mod 22.
 - (c) (3 points) Solve the congruence $15^x \equiv 3 \pmod{22}$.

–Amin Witno