

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 1

Number Theory

18–11–2018

Choose 5 problems to solve. No bonus.

- Evaluate $\gcd(1008, 540)$ using prime factorization.
 - Evaluate $\gcd(1008, 540)$ using euclidean algorithm (mod operation).
- Find all the integer solutions to the linear equation $343x + 231y = 42$.
 - Use (a) to write the solution class to the congruence $343x \equiv 42 \pmod{231}$.
- Write Wilson's theorem for $p = 103$.
 - Use (a) to compute $99! \% 103$.
- Prove that $n^4 \equiv n^2 \pmod{4}$ for all $n \in \mathbb{Z}$.
- Prove this theorem: If $a \mid bc$ and $\gcd(a, c) = 1$, then $a \mid b$.
- Prove that if p is a prime and $m^2 \equiv 49 \pmod{p}$, then $m \in [7]_p$ or $m \in [-7]_p$.

–Amin Witno