**Exam 2**                    **Number Theory**                    03–01–2019

Choose 5 problems out of 6. No bonus.

1. This problem has 4 parts:

   (a) Evaluate $|5|_{13}$.

   (b) Find all the primitive roots mod 11.

   (c) If $g$ is a primitive root mod 27, evaluate $|g^{15}|_{27}$.

   (d) Count how many primitive roots exist mod 37.

2. Evaluate $11^{70} \% 100$ using successive squaring algorithm.

3. Find all the integer solutions of $x^{13} \equiv 7 \pmod{48}$.

4. This problem has 2 parts:

   (a) Prove that 3 is a primitive root mod 14.

   (b) Use (a) to find all the integer solutions of $5^x \equiv 13 \pmod{14}$.

5. Prove that $a^{37} \equiv a \pmod{95}$ for all integers $a$. (Use the Chinese remainder theorem and Fermat's little theorem.)

6. Prove that $\phi(n^2) = n\phi(n)$ for all positive integers $n$.

–Amin Witno