

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2

Numerical Analysis

24–12–2006

Choose five problems.

1. Let $x_0 = 0$, $x_1 = 0.5$, and $x_2 = 1$. Construct interpolation polynomial of degree two to approximate $f(0.45)$ where $f(x) = \tan x$.
2. Use Neville's method to approximate $\sqrt{7}$ with the function $f(x) = 7^x$ and the values $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$.
3. Determine each missing entry in the following table using the three-point formulas with $h = 0.1$ or $h = -0.1$.

x	$f(x)$	$f'(x)$
1.4	0.925212	
1.5	0.778073	
1.6	0.549355	

4. Approximate the integral

$$\int_0^3 e^{x^2} dx$$

- (a) using the Midpoint rule
- (b) using Simpson's Three-Eighths rule

5. Approximate the integral

$$\int_0^2 \frac{1}{x^2 + 2} dx$$

with $n = 4$

- (a) using the Composite Trapezoidal rule
- (b) using the Composite Simpson's rule

6. The actual value of the integral

$$\int_0^{\pi/4} \cos^2 x dx$$

is 0.64269908. Compute the Simpson's rule approximations $S(a, b)$, $S(a, \frac{a+b}{2})$, and $S(\frac{a+b}{2}, b)$ to verify the error estimation

$$\frac{1}{15} \left| S(a, b) - S\left(a, \frac{a+b}{2}\right) - S\left(\frac{a+b}{2}, b\right) \right|$$

comparing it to the actual error.