Philadelphia University

Department of Basic Sciences

Final Exam **Numerical Analysis** **29–1–2007**

There are 10 problems in this exam.
First, delete 3 problems of your choice. (1 mark)
Then solve the remaining 7 problems. (7 marks each)

1. Use Runge-Kutta of order four to approximate the solution for w_1 (only) to the following initial-value problem, and compare the results to the actual value.

$$y' = 1 + (t-y)^2, \quad 3 \leq t \leq 4, \quad y(3) = 2.5, \quad h = 0.5$$

with actual solution $y(t) = t + \frac{1}{1-t}$.

2. Use (a) Modified Euler method and (b) Taylor method of order two to approximate the solution for w_1 (only) to the following initial-value problem, and compare the results to the actual value.

$$y' = 1 + (t-y)^2, \quad 3 \leq t \leq 4, \quad y(3) = 2.5, \quad h = 0.5$$

with actual solution $y(t) = t + \frac{1}{1-t}$.

3. Use Euler method to approximate the solution for w_1 (only) to the following initial-value problem, and then estimate the error bound.

$$y' = y - 2t^2 + 2, \quad 0 \leq t \leq 2, \quad y(0) = 1, \quad h = 0.25$$

with actual solution $y(t) = 2(t+1)^2 - e^t$.

4. Approximate the integral with $h = 0.5$

$$\int_0^2 x^2 e^{-x^2}\, dx$$

using (a) Composite Trapezoidal rule, (b) Composite Simpson's rule, and (c) Composite Midpoint rule.

5. Approximate the following integral using (a) Trapezoidal rule and (b) Midpoint rule and find a bound for the error for each method.

$$\int_{1}^{1.5} x^2 \ln x \, dx$$

6. Use (a) forward/backward-difference formulas and (b) three-point formulas to determine each missing entry in the following table.

x	$f(x)$	$f'(x)$
0.2	0.74140	
0.4	1.37180	
0.6	1.90211	

7. Use Neville's method to approximate $\sqrt{5}$ with the function $f(x) = 5^x$ and the values $x_0 = -1$, $x_1 = 0$, $x_2 = 1$, and $x_3 = 2$.

8. Consider the sequence

$$p_n = \frac{1}{n^3}\,, \quad n \geq 1$$

   (a) Show that the sequence converges linearly to $p = 0$.

   (b) How large must n be before $|p_n - p| \leq 5 \times 10^{-5}$?

   (c) Generate the first three terms of the sequence $\{\hat{p}_n\}$ using Aitken's Δ^2 method.

9. (a) Use the Bisection method to find a solution for $f(x) = x^5 - 2x^3 - 3 = 0$ (up to p_5) on the interval $[1, 2]$.

   (b) Determine the number of iterations needed to achieve an approximation with accuracy 5×10^{-7}.

10. (a) Use Newton's method to find a solution (up to p_3) for $f(x) = x - \cos x = 0$ on the interval $[0, \pi/2]$ using $p_0 = 0$.

   (b) Repeat (a) using the Secant method, with $p_0 = 0$ and $p_1 = \pi/2$.

–Amin Witno