Philadelphia University
Department of Basic Sciences

Final Exam **Probability Theory** **29–01–2020**

Each problem is worth 4 points.

1. Two dice are rolled. Let (x, y) denote the outcome. Let $A = \{(x, y) \mid x + y \geq 6\}$ and $B = \{(x, y) \mid xy \text{ is odd}\}$. Compute $P(A \cup B)$.

2. A Mathematics student has probability 81% to pass the Final Exam if he will study, and 54% if he will NOT study. The probability he will study is 66%. Given that the student passed the Final Exam, compute the probability that he did NOT study.

3. Compute $P(6 < X < 9)$ given the distribution function
$$F(x) = \begin{cases} 1 - \frac{9}{x^2} & \text{for } x \geq 3 \\ 0 & \text{for } x < 3 \end{cases}$$

4. Given the joint probability distribution $f(x, y) = k(x^2 + y)$ on the domain $x \in \{-1, 1, 3\}$ and $y \in \{2, 3\}$. (a) Find the value of k. (b) Compute $P(X + Y > 3)$.

5. Compute $P(1 < X < 2;\ Y \leq 2)$ given the joint distribution function
$$F(x, y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y} & \text{for } x, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

6. Compute the conditional density of Y given $(X = 1)$ with the joint p.d.f
$$f(x, y) = \begin{cases} \frac{1}{5}(2x + y) & \text{for } 0 < x < 2;\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

7. Write the double integral (ONLY THE INTEGRAL) to compute $P(X, Y < \frac{1}{2})$ given the joint p.d.f $f(x, y)$ on the domain $\{0 < x < 1;\ -x < y < x\}$.

8. Given that $\sigma_X^2 = 3$, $\sigma_Y^2 = 4$, $\sigma_Z^2 = 5$ and $\sigma_{XY} = 3$, $\sigma_{XZ} = -2$, $\sigma_{YZ} = 1$. Let $W = X + 2Y - 3Z$ and compute the variance σ_W^2.

9. Compute the covariance σ_{XY} given the joint probability density function
$$f(x, y) = \begin{cases} x + y & \text{for } 0 < x, y < 1 \\ 0 & \text{otherwise} \end{cases}$$

10. Compute μ and σ^2 for each distribution.
    (a) The uniform distribution given by $f(x) = \frac{1}{b-a}$ on the domain $x \in [a, b]$.
    (b) The Pareto distribution given by $f(x) = \frac{2}{x^3}$ on the domain $x \in [1, \infty)$.

11. (BONUS) Let X have the exponential distribution with the probability density function $f(x) = \lambda e^{-\lambda x}$ on the domain $x \in [0, \infty)$, for some $\lambda > 0$. Compute μ.

–Amin Witno