

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

Exam 2

Set Theory

7-5-2006

Each problem is worth 4 points.

1. Prove by induction for all  $n \geq 1$ .

$$\sum_{k=0}^{n-1} 9^k = \frac{9^n - 1}{8}$$

2. Prove that  $A \oplus B = A - B$  if and only if  $B \subseteq A$ .
3. Let  $A = \{1, 2, 3, 4\}$ . Give an example of a relation  $R$  on  $A$  which is
- (a) symmetric, transitive, not reflexive
  - (b) reflexive, not anti-symmetric, not transitive
4. Let  $A = \{2, 3, 6, 8, 12, 24\}$  and  $R = \{(a, b) \in A \times A \mid b \bmod a = 0\}$ .
- (a) Find the elements of  $R$ .
  - (b) Prove that  $R$  is a partial order relation.
  - (c) Is  $R$  a total order? Why or why not?
5. Let  $\mathbf{Z}$  be the set of integers and  $R = \{(a, b) \in \mathbf{Z} \times \mathbf{Z} \mid a + b \text{ is even}\}$ .
- (a) Prove that  $R$  is an equivalence relation.
  - (b) Find the equivalence classes of  $\mathbf{Z}$  under  $R$ .

-Amin Witno