Philadelphia University

Department of Basic Sciences

Final Exam **Set Theory** **5–6–2006**

1. (a) Prove $p \to q \equiv \neg p \vee q \equiv \neg q \to \neg p$ using truth table.
   (b) What is the contrapositive of the proposition "If x is an integer then x is even"?
   (c) Suppose $P(x, y) : x^2 + y^2 = (x + y)^2$. What is the value of the proposition $\exists! x \forall y P(x, y)$?
   (d) What is the negation of the proposition "Every integer is even"?
   (e) What is the negation of the proposition "There is a unique rational number x such that $\sqrt{x}$ is irrational"?

2. (a) What is the proposition used in the proof by contrapositive?
   (b) What is the proposition used in the proof of equivalent statement?
   (c) What is the proposition used in the proof by cases?
   (d) Let x be a rational number and let y be an irrational number. Prove that $x + y$ is irrational, using proof by contradiction.
   (e) Prove by induction, $2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$ for all $n \geq 1$.

3. (a) Suppose A and B are sets. What is the definitions of $A - B$ and $A \oplus B$?
   (b) What is the definition of $S \subseteq A$?
   (c) Suppose $S \subseteq A$. Prove that $A \oplus S = A - S$.
   (d) Suppose A is a set. What is the definition of $P(A)$?
   (e) Suppose $S \subseteq A$. Prove that $P(S) \subseteq P(A)$.

4. (a) Suppose R is an equivalence relation on a set A. What is the definition of the equivalence class of $x \in A$?
   (b) What is the difference between a partial order relation and a total order relation?
   (c) Let $A = \{1, 2, 3\}$. Give an example of a partial order relation on A which is not a total order.
   (d) What is the definition of a well ordering?
   (e) Suppose R is a well ordering on a set A. Prove that R is a total order relation.

5. (a) What is the definition of a function from A to B?
   (b) What is the definition of a one-to-one function?
   (c) What is the definition of an onto function?
   (d) Suppose $f : A \to B$ is a function. What is the definition of f^{-1}?
   (e) Suppose $f : A \to B$ is a one-to-one and onto function. Prove that f^{-1} is a function from B to A.

6. (a) Suppose A and B are sets. What is the definition of $|A| = |B|$?
   (b) Let $A = \{2, 3, 4, 5, \ldots\}$ and $B = \{3, 4, 5, 6, \ldots\}$. Prove that $|A| = |B|$.
   (c) What is the definition of a countable set?
   (d) True or False? If A and B are both infinite countable sets then $|A| = |B|$.
   (e) Give two examples of uncountable sets A and B such that $|A| \neq |B|$.

–Amin Witno