

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2

Set Theory

09–05–2010

Solutions must be complete in order to receive full credit.

1. Let $x \in \mathbb{Z}$. Prove that $3x^2 - 2x + 5$ is odd if and only if x is even.
2. Prove that $2^{3n} + 13$ is a multiple of 7 for all integer $n \geq 0$. Use induction.
3. Let $a \in \mathbb{Q}$ and b an irrational number. Prove that $a + b$ is irrational. Use contradiction.
4. Let $A = \{x \in \mathbb{Z} \mid 1 \leq x \leq 10\}$ and R be a relation on A such that $(a, b) \in R$ if and only if $(a - b)$ is a multiple of 3. Prove that R is an equivalence relation and find the equivalence classes.

–Amin Witno