

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam

Set Theory

24-01-2011

Solutions must be complete in order to receive full credit.

1. Prove the equivalence statement for $x \in \mathbb{Z}$.

The number $2x^2 - 7x + 1$ is odd if and only if x is even.

2. Prove using induction for all $n \in \mathbb{N}$.

$$1 + 4 + 9 + 16 + \dots + n^2 = \frac{2n^3 + 3n^2 + n}{6}$$

3. Prove using truth table or Venn diagrams.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

4. Consider the matrix for a relation R :

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- (a) Is R reflexive? Why or why not?
(b) Is R symmetric? Why or why not?
(c) Is R anti-symmetric? Why or why not?
(d) Is R transitive? Why or why not?
(e) Find $R^2 = R \circ R$.
(f) Find the matrix for R^2 .
5. Let $A = \{x \in \mathbb{Z} \mid 1 \leq x \leq 9\}$ and $R = \{(a, b) \mid b \bmod a = 0\} \subseteq A \times A$.
- (a) Prove that R is a partial order relation.
(b) Draw the digraph of R and its Hasse diagram.
(c) Is R a total order? Why or why not?
(d) Is there a least element? Why or why not?
6. Let A be the set of all positive odd numbers. Prove that $|A| = \aleph_0$.