Exam 1   **Set Theory**   06–11–2012

Part 1: Short Answer

1. Which one is not equivalent to $p \leftrightarrow q$? Choose one:
   (a) $p \oplus \neg q$ (b) $\neg p \oplus q$ (c) $\neg p \leftrightarrow \neg q$ (d) $\neg p \oplus \neg q$

2. Which one is equivalent to $\neg p \to q$? Choose one:
   (a) $p \to \neg q$ (b) $q \to \neg p$ (c) $\neg q \to p$ (d) $\neg q \to \neg p$

3. Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. What is $(A \cup B) - B$?

4. Let $A = \{1, 2, 3, 4, 5, 6\}$. What are the elements of the set $\{x \in \mathbb{Z} \mid 2x \in A\}$?

5. Let $A \subseteq B$. Then $A \oplus B =$ (Choose one:)
   (a) $A \cap B$ (b) $A - B$ (c) $B - A$ (d) $A \cup B$

6. Let $A = \{\phi, x\}$. What is $P(A)$?

7. Let $A = \{1, 2\}$ and $B = \{2, 3, 4\}$. What is $|P(A \times B)|$?

8. Which one is false? Choose one:
   (a) $x \in \{x, y, z\}$ (b) $\{x\} \subseteq \{x, y, z\}$ (c) $\phi \subseteq \{x, y, z\}$ (d) $\phi \in \{x, y, z\}$

Part2: Complete Solution

1. Prove the equivalence $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$.

2. Use proof by cases to prove that $3x^2 - x - 7$ is an odd number for all $n \in \mathbb{Z}$.

3. Prove using contrapositive: If $x^2 + x$ is an irrational number, then $2x + 1$ is also irrational.

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