

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

**Exam 1**

**Set Theory**

**25-03-2013**

1. Find the elements:

- (a)  $\{-1, 0, 1, 3, 5, 7\} \oplus \{1, 2, 3, 4, 5\}$
- (b)  $\{x \in \mathbb{Z} \mid 0 \leq x < 9\} - \{1, 2, 3, 4, 5\}$
- (c)  $\{x \in \mathbb{N} \mid x^2 > 10\} \cap \{1, 2, 3, 4, 5\}$
- (d)  $\{x \in \mathbb{Q} \mid 2x \in \mathbb{Z}\} \cap \{x \in \mathbb{R} \mid x^2 \leq 1\}$

2. Prove the equivalence:

$$(p \rightarrow q) \wedge (r \rightarrow q) \equiv (p \vee r) \rightarrow q$$

- 3. Use direct proof to prove that if  $x$  is an odd number, then  $(x + 3)^2 - 23$  is also odd.
- 4. Use contrapositive to prove that if  $x^2 - 1$  is an irrational number, then  $x - 1$  is also irrational.
- 5. Use proof by cases to prove that  $x^3 + 3x$  is an even number for any  $x \in \mathbb{Z}$ .

-Amin Witno