Philadelphia University

Department of Basic Sciences

Final Exam **Set Theory** **08–06–2013**

Write complete solutions.

1. Prove the equivalence $p \oplus q \equiv (p \wedge \neg q) \vee (q \wedge \neg p)$.
2. Use contrapositive to prove that for $x \in \mathbb{Z}$, if $3x^2 - 7x - 4$ is odd, then x is even.
3. Use contradiction to prove that the number $\log_{10} 5$ is irrational.
4. Use induction to prove that $3^n > 1 + 2^n$ for all integer $n \geq 2$.
5. Let $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = -3x + 5$. Prove that f is one-to-one and onto.
6. For any sets A and B, let $(A, B) \in R$ if and only if there exists a one-to-one and onto function $f : A \to B$. Prove that R is an equivalence relation.
7. Let $S = \{n \in \mathbb{Z} \mid n \geq -5\}$. Prove that $|S| = \aleph_0$.

–Amin Witno