Philadelphia University

Department of Basic Sciences

Exam 2 **Set Theory** **11–12–2013**

1. Prove:
   (a) There is a prime number p such that $p \bmod 3 = 2$ and $p \bmod 4 = 3$.
   (b) Not all natural number $2n + 1$ is prime.
2. Use contradiction to prove that $\sqrt[3]{4}$ is an irrational number.
3. Use induction to prove that $11^n - 6$ is a multiple of 5 for all $n \in \mathbb{N}$.
4. Let $R = \{(a, b) \in \mathbb{R} \times \mathbb{R} \mid |a| = |b|\}$.
   (a) Prove that R is an equivalence relation on $\mathbb{R}$.
   (b) Find the equivalence class of $a = 3$.
5. Let $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid ab > 0\}$. True or false?
   (a) R is reflexive.
   (b) R is symmetric.
   (c) R is anti-symmetric.
   (d) R is transitive.

–Amin Witno