Philadelphia University

Department of Basic Sciences

Final Exam **Set Theory** **05–02–2014**

Solutions must be complete in order to receive full credit.

1. Short answer:
   (a) Let $A = \{x \in \mathbb{N} \mid x \leq 9\}$ and $B = \{x \in \mathbb{N} \mid x < 6\}$. Find the elements in $A \oplus B$.
   (b) Let $A = \{x \in \mathbb{Q} \mid 2x \in \mathbb{Z}\}$ and $B = \{x \in \mathbb{Q} \mid 0 \leq x \leq 2\}$. Find the elements in $(A \cap B) - \mathbb{Z}$.
   (c) Let $A = \{\{a\}, \phi\}$. What is the power set $P(A)$?
   (d) Let $f : \mathbb{N} \to \mathbb{Z}$ such that $f(x) = x \bmod 3$. What is the range of f?
   (e) Let $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = x^2 + 1$, and let $S = \{x \in \mathbb{R} \mid -1 \leq x \leq 10\}$. What is the set $f^{-1}(S)$?
2. Let $x \in \mathbb{Z}$. Prove that $x^2 - 1$ is a multiple of 4 if and only if x is odd.
3. Use contradiction to prove that the number $\log 5$ is irrational.
4. Use induction to prove the following formula for all $n \in \mathbb{N}$.
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$
5. Let $f : \mathbb{Q} \to \mathbb{Q}$ such that $f(x) = 3x - 2$. Prove that the function f is one-to-one and onto.
6. Let A and B represent any sets. Use the definition of cardinal numbers to prove that the relation $R = \{(|A|, |B|) \mid |A| = |B|\}$ is an equivalence relation.
7. Use the definition of cardinal numbers to prove that $|\mathbb{Z} - \mathbb{N}| = \aleph_0$.
8. If A and B are countable sets, prove that $A \times B$ is also countable.

–Amin Witno