

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam

Set Theory

10-06-2014

Solutions must be complete in order to receive full credit.

1. Find the elements in A .
 - (a) $A = \{1, 2, 3, 4, 5\} \oplus \{1, 3, 5, 7\}$
 - (b) $A = \{x \in \mathbb{Z} \mid x^2 + 2x - 3 = 0\} - \mathbb{N}$
 - (c) $A = \{x \in \mathbb{R} \mid x^2 \leq 9\} \cap \mathbb{Z}$
 - (d) $A = \{X \subseteq \{1, 3, 5\} \mid |X| = 2\}$
2. Let $x \in \mathbb{Z}$. Prove that the number $3x^2 - 2x - 11$ is odd if and only if x is even.
3. Use contradiction to prove that the number $\sqrt{2}$ is irrational.
4. Use induction to prove the following formula for all $n \in \mathbb{N}$.
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$
5. Let $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid a - b \text{ is a multiple of } 3\}$. Prove that R is an equivalence relation on \mathbb{Z} .
6. Let $A = \{x \in \mathbb{Z} \mid x > -9\}$. Prove that $|A| = \aleph_0$.
7. Let $A = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$ and $B = \{x \in \mathbb{R} \mid 1 \leq x \leq 3\}$. Prove that $|A| = |B|$.

-Amin Witno