Philadelphia University
Department of Basic Sciences and Mathematics

Final Exam — Set Theory — 2-2-2015

Name: ________________ Number: ________________ Section: ________________

1. (16 points) Circle the correct answer.

   (a) The proposition $p \to (q \to r)$ is equivalent to

   (A) $q \to (p \to r)$ (B) $r \to (q \to p)$ (C) $(p \to q) \to r$ (D) $(q \to r) \to p$

   (b) If $A \subseteq B$, then $A \oplus B$ equals

   (A) $A \cup B$ (B) $A \cap B$ (C) $A - B$ (D) $B - A$

   (c) Let $A = \{2,5,6,7,9\}$ and the equivalence relation $R = \{(a,b) \mid a \bmod 2 = b \bmod 2\}$. Find the equivalence classes.

   (A) $\{2,6\},\{5,7,9\}$ (B) $\{2,4,8\},\{5,7\}$ (C) $\{2,7,9\},\{5,6\}$ (D) $\{2,7,8\},\{4,5\}$

   (d) Which of the following relations is transitive but not anti–symmetric?

   (A) $\{(1,3),(3,1),(1,1),(4,4)\}$ (B) $\{(1,3),(2,1),(2,3),(4,4)\}$
   (C) $\{(1,3),(2,3),(2,4),(3,4)\}$ (D) $\{(1,3),(3,1),(1,1),(3,3)\}$

   (e) What is the negation of the proposition "Every integer is even" ?

   (A) Every integer is odd (B) There exists an even integer
   (C) There exists an odd integer (D) All the integers are even

   (f) Let $f: \mathbb{N} \to \mathbb{Z}$ such that $f(x) = x \bmod 3$. What is the range of f?

   (A) $\mathbb{N}$ (B) $\mathbb{Z}$ (C) $\{0,1,2,3\}$ (D) $\{0,1,2\}$

   (g) Find $|A|$ if $A = \left\{X \in P(\{1,2,3\}) \,\middle|\, |X| = 2\right\}$

   (A) 3 (B) 8 (C) 0 (D) 2

   (h) Which of the following is an infinite set?

   (A) $\left\{x \in \mathbb{R} \mid x^2 + 1 = 0\right\}$ (B) $\left\{x \in \mathbb{N} \mid x^2 + x \text{ is prime}\right\}$
   (C) $\left\{1,3,5\right\} \times \left\{2,4,6\right\}$ (D) The rational numbers in $(0,1)$

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2. (5 points) Let $A = \left\{ x \in \mathbb{Z} \mid x > -9 \right\}$. Prove that $|A| = |\mathbb{N}|$.

3. (5 points) Let $x \in \mathbb{Z}$. Prove that $x^2 - 1$ is a multiple of 4 if and only if x is odd.

4. (4 points) Use the contradiction to show that $\sqrt{2}$ is irrational.

5. (4 points) Use mathematical induction to prove that $2^n < n!$ for all $n \geq 4$.

6. (6 points) Let A and B represents any sets. Use the definition of cardinal numbers to prove that the relation $R = \left\{ (|A|, |B|) \,\middle|\, |A| = |B| \right\}$ is an equivalence relation.