Exam 2   **Set Theory**   **06–05–2015**

Each problem is worth 4 points.

1. Translate and prove:

   (a) There is a natural number x such that $x \bmod 4 = 2$ and $x \bmod 5 = 3$.

   (b) Not all real numbers x satisfy $x^2 \geq x$.

2. Prove that the number $\log_{10} 8$ is irrational, using contradiction.

3. Use induction to prove that $5^{2n} - 4$ is a multiple of 3 for all natural numbers n.

4. Let $A = \{x \in \mathbb{Z} \mid 1 \leq x \leq 5\}$ and $R = \Big\{(a,b) \in A \times A \mid a + b > 2\Big\}$.

   (a) Is R reflexive? (True or false?)

   (b) Is R symmetric? (True or false?)

   (c) Is R anti-symmetric? (True or false?)

   (d) Is R transitive? (True or false?)

5. Let $R = \Big\{(x,y) \in \mathbb{N} \times \mathbb{N} \mid x \text{ is a multiple of } y\Big\}$. Prove that R is a partial order relation.

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