

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

Final Exam

Set Theory

14-06-2015

Choose 8 problems; 5 points each.

1. Prove the equivalence statement  $P \rightarrow (Q \wedge R) \equiv (P \rightarrow Q) \wedge (P \rightarrow R)$ .
2. Use proof by cases to prove that the number  $5x^2 - 7x - 3$  is odd for any  $x \in \mathbb{Z}$ .
3. Use contradiction to prove that the number  $\sqrt{2}$  is irrational.
4. Use induction to prove that  $3^n > 1 + 2^n$  for all integer  $n \geq 2$ .
5. Use induction to prove the following formula for all  $n \in \mathbb{N}$ .

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

6. Short answers:

- (a) Let  $A = \{n \in \mathbb{Z} \mid -2 \leq n \leq 5\}$  and  $B = \{n \in \mathbb{N} \mid n \geq 7\}$ . Find the elements in the set  $(A \oplus \mathbb{N}) - B$ .
  - (b) Let  $A = \{1, 2, 3, 4\}$  and  $S = \{X \in P(A) \mid |X| = 3\}$ . Find the elements of  $S$ .
  - (c) Let  $A = \{n \in \mathbb{N} \mid n < 11\}$  and consider the equivalence relation  $R = \{(x, y) \mid x \bmod 3 = y \bmod 3\}$  on  $A$ . Find the elements in the equivalence class  $[7]$ .
  - (d) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x) = x^2$ . For the closed interval  $S = [-5, 2] \subseteq \mathbb{R}$ , find the set  $f^{-1}(S)$ .
  - (e) Give one example of a function  $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$  that is one-to-one and onto.
7. Let  $A$  and  $B$  represent any sets. Use the definition of cardinality to prove that the relation  $R = \{(|A|, |B|) \mid |A| = |B|\}$  is an equivalence relation.
  8. Let  $A = \{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$  and  $B = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$ . Prove that  $|A| = |B|$ .
  9. Prove that  $|\mathbb{Z} - \mathbb{N}| = \aleph_0$ .